INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH, BHOPAL

END-SEMESTER EXAMINATION
TOTAL TIME : 3 HOURS

ACADEMIC YEAR: 2018-19 **COURSE: ECO311**
INSTRUCTOR: PARIKSHIT DE **PART: I**

▶ *Answer the following questions:*$(1 \times 10 = 10)$

Q 1. Consider an indivisible object is to be allocated among three agents namely, $\{1, 2, 3\}$. 1 wins the object if and only if reported valuation of 1 is greater than or equal to average of reported valuation of 2 and 3; otherwise the agent (other than 1) with highest valuation wins. What payment rule would implement such an allocation in dominant strategy? Can such payment rule be feasible? Can such allocation rule be affine maximizer allocation rule? Explain. (4+3+3)

INSTRUCTOR: ABHINABA LAHIRI **PART: II**

▶ *Answer all the following questions:*$(4 \times 10 = 40)$

Q 1. Consider a profile of single peaked preferences $P = (P_1, \ldots, P_n)$, where n is an odd number of agents. For every pair of alternatives $a, b \in A$, we say a beats b at P if

$$|\{i \in N : aP_ib\}| > |\{i \in N : bP_ia\}|$$

It is known that at every single peaked preference profile P, there will always exist an alternative x such that x beats y at P for every other alternative y. We call such an alternative the winner at P and denote it as $\omega(P)$. Consider the social choice function f which picks $\omega(P)$ at every single peaked preference profile P. Show that f is strategy-proof, unanimous, and anonymous. (10)

Q 2. Let A be a finite set of alternatives and $\succ$ be a linear order over A. Suppose $a_L, a_R \in A$ be two alternatives such that $a \succ a_L$ for all $a \in A\{a_L\}$ and $a_R \succ a$ for all $a \in A\{a_R\}$ - in other words, a_L is the left-most alternative and a_R is the right-most alternative with respect to $\succ$. Let $\mathbb{D}$ be the set of all possible single-peaked strict orderings over A with respect to $\succ$. An SCF $f : \mathbb{D}^n \longrightarrow A$ maps the set of preference profiles of n agents to A.
Let $P_i(1)$ denote the peak of agent i in P_i. Suppose f satisfies the following property(call it property π). There is an alternative $a^\star \in A$ such that for any preference profile $(P_1, \ldots, P_n) \in \mathbb{D}^n$, where $P_i(1) \in \{aL, aR\}$ for all $i \in N$ with at least one agent's peak at a_L and at least one agent's peak at a_R, $f(P_1, \ldots, P_n) = a^\star$.

a. Suppose f is strategy-proof, efficient, anonymous, and satisfies property π. Then, give a precise (simplified) description of f (using $a^\star$), i.e., for every preference profile P, what is $f(P)$? (5)

b Can f be strategy-proof, anonymous, and satisfy property π, but not efficient (give a formal argument or an example)? (5)

Q 3. Let A be a finite set of alternatives and $f : \mathbb{P}^n \longrightarrow A$ be a social choice function that is unanimous and strategy-proof. Suppose $|A| \geq 3$. Now, consider another social choice function $g : \mathbb{P}^2 \longrightarrow A$ defined as follows. The scf g only considers profiles of two agents, denote these two agents as 1 and 2. For any $(P_1, P_2) \in \mathbb{P}^2$, let

$$g(P_1, P_2) = f(P_1, P_2, P_1, P_1, ..., P_1),$$

i.e., the outcome of g at (P_1, P_2) coincides with the outcome of f at the profile where agents 1 and 2 have preferences P_1 and P_2 respectively, and all other agents have preferences P_1. Show that g is a dictatorship scf. (10)

Q 4. Consider the following matching problem between hospitals and doctors, where some doctors may form a couple. Let $H = \{h_1, h_2\}$ denote the set of hospitals. Each hospital has capacity 2, i.e., $k_{h_1} = k_{h_2} = 2$. Let $D = \{d_1, d_2, m, f\}$ denote the set of doctors, where the only couple is $c = \{f, m\}$. Their preferences are given below.

P_{h_i}	$\overline{P_{h_i}}$	P_{d_1}	P_{d_2}	P_f	P_m	P_c
f	$\{f, d_1\}$	h_1	h_1	h_2	h_1	(h_2, h_1)
d_1	$\{f, d_2\}$	h_2	h_2	h_1	h_2	(h_1, h_1)
d_2	$\{f, m\}$					(h_2, h_2)
m	$\{d_1, d_2\}$					(h_1, h_2)
	$\{d_1, m\}$					
	$\{d_2, m\}$					

In the above table, P_{h_i} denotes the preferences of hospitals over the set of individual doctors. $\overline{P_{h_i}}$ denotes the preferences of each hospital extended to pairs of doctors. Each hospital would like to fill their capacity rather than keeping a position vacant. P_{d_1}, P_{d_2}, P_f and P_m denotes the individual doctors preferences over hospitals. P_c denote the couple's preference. The couple's preference over pairs of hospitals, where one member is matched and the other one is unmatched, is not shown in the table, but assumed to be ranked below the shown pairs. Another point to note that in couple's preference the alternative (h_1, h_2) denotes the situation where f gets matched to h_1 and m gets matched to h_2. Explain why given the preferences in the above table, every possible matching where hospitals operate at full capacity will be blocked by some blocking coalition. (10)