

Economic Theory: Mechanism Design (ECO 311/611)

Assignment

Total Marks : 30 Points

1. Consider the unanimous SCF $f : \mathbb{P}^n \rightarrow A$ defined as follows. If $P_1(1) = \dots = P_n(1) = a$, then $f(P_1, \dots, P_n) = a$. Else, $f(P_1, \dots, P_n) = b$ for some alternative $b \in A$. In other words, f satisfies unanimity wherever possible and picks a “status-quo” alternative b otherwise. Argue how f can be manipulated if there are at least three alternatives? **10 points**
2. Let the number of alternatives be m . Show that the number of single-peaked preference orderings with respect to \succ (an exogenously given ordering of alternatives) is 2^{m-1} . **10 points**
3. Let $A = \{a_1, a_2, \dots, a_m\}$ be the set of m alternatives and N be the set of n agents. Let \succ be an exogenously given ordering of alternatives such that $a_m \succ a_{m-1} \succ \dots \succ a_1$. Let \mathbb{D} be the set of all strict single peaked preferences defined over A with respect to \succ . Consider the following SCF $f : \mathbb{D}^n \rightarrow A$:

$$f(P) = \begin{cases} a_1 & \text{if } |\{i \in N : a_1 P_i a_m\}| \geq |\{i \in N : a_m P_i a_1\}| \\ a_m & \text{otherwise} \end{cases}$$

Show that f is strategy-proof but not Pareto optimal.

10 points