Game Theory (ECO307)

September 25, 2018

Instructions: Answer all questions.
Duration: 2 hours
Total Marks: 100

1 (20 marks). Define a pure strategy Nash equilibrium in a normal form game. Find all pure strategy Nash equilibria of the game described below:

		Player 2	
		L	R
Player 1	T	2, 1	0, 0
	B	1, 2	3, 0

2 (20 marks). Consider an infinite period Rubinstein's bargaining model. The timing of the bargaining game is as follows:

(1*a*) At the beginning of the first period, player 1 proposes to take a share s_1 of the dollar, leaving $1 - s_1$ for player 2.

(1*b*) Player 2 either accepts the offer (in which case the game ends and the payoffs s_1 to player 1 and $1 - s_1$ to player 2 are immediately received) or rejects the offer (in which case play continues to the second period).

(2*a*) At the beginning of the second period, player 2 proposes that player 1 take a share s_2 of the dollar, leaving $1 - s_2$ for player 2.

(2*b*) Player 1 either accepts the offer (in which case the game ends and the payoffs s_2 to player 1 and $1 - s_2$ to player 2 are immediately received) or rejects the offer (in which case the play continues to the third period).

(3) The sequence of game described above is repeated infinite number of times.

Suppose the players in Rubinstein's infinite-horizon bargaining game have different discount factors: δ_1 for player 1 and δ_2 for player 2. Show that in the backwards-induction outcome, player 1 offers the settlement

$$\left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \ \frac{\delta_2 (1 - \delta_1)}{1 - \delta_1 \delta_2} \right)$$

to player 2, who accepts.

3 (20 marks). The simultaneous move game (below) is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. Can the payoff (4, 4) be achieved in the first stage in a pure-strategy subgame perfect Nash equilibrium? If so, give strategies that do so.

		Player 1		
		L	C	R
Player 2	T	3, 1	0, 0	5, 0
	M	2, 1	1, 2	3, 1
	B	1, 2	0, 1	4, 4

4 (20 marks). Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Marginal and fixed cost of production is zero for each firm. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively. What is the subgame-perfect outcome of this game?

5 (20 marks). Show that in the $n-$player normal-form game $G = \{S_1, ..., S_n;\ u_1, ..., u_n\}$, if the strategies $(s_1^*, ..., s_n^*)$ are a Nash equilibrium, then they survive iterated elimination of strictly dominated strategies.